EEE4086F
RF and Microwave systems

Transmission Lines and Microwave Networks

Course news

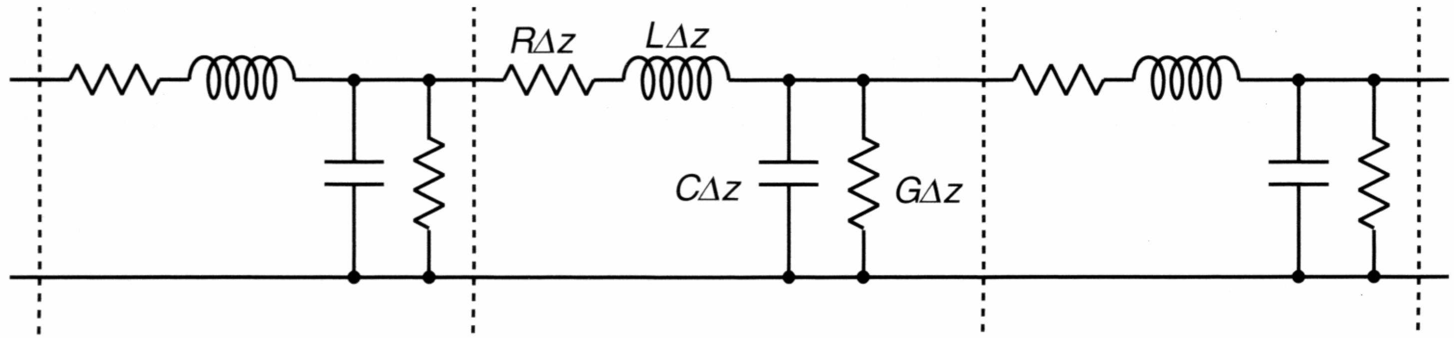
- Website created in http://rrsg.ee.uct.ac.za/EEE4086F
- First practical Wednesday 27/02 at 11h00. Subject can be downloaded from the website
- Essay's on regulation must be completed for Thursday 13/03.
- Read chapter 1 and 2 of the Pozar book for Thursday's Lecture (28/02)

Introduction

- **Circuit analysis is a simplification of Maxwell's equations using the assumption that the physical dimensions of the network are much smaller than the electrical wavelength (d<<λ).**
- **But as frequency increases, wavelength decreases (since all EM waves propagate at the speed of light), and this assumption becomes false.**
- **The transmission line is a distributed-parameter network, where voltages and currents can vary in magnitude and phase over the length of the line.**

Lumped Element Model

- The transmission line is often represented as a two-wire line
- The transmission line is divided into segments of length Δz, which are modeled by the same lumped element model.
- The units of the circuit parameters are measured per unit length and the voltage and current can therefore vary along the length of the line.



R: series resistance (Ω/m) **L: series inductance (H/m)**
G: shunt conductance (S/m) **C: shunt capacitance (F/m)**

Transmission Line Equations

- Kirchoff's Voltage and Current law can be applied to the lumped circuit model, giving 2 equations involving current and voltage at either end of the differential line segment.
- In the limit as z $\rightarrow$ 0, one obtains a set of partial differential equations in the time domain

$$
\begin{aligned}
\frac{\partial v(z,t)}{\partial z} &= -Ri(z,t) - L\frac{\partial i(z,t)}{\partial t} \\
\frac{\partial i(z,t)}{\partial z} &= -Gv(z,t) - C\frac{\partial i(z,t)}{\partial t}
\end{aligned}
$$

- These equations can be transformed to the frequency domain to give the ordinary differential equations:

$$
\begin{aligned}
\frac{dV(z)}{dz} &= -(R + j\omega L)I(z) \\
\frac{dI(z)}{dz} &= -(G + j\omega C)V(z)
\end{aligned}
$$

RF and Microwave Systems

Transmission Lines

Smith Chart

Microwave Networks

Impedance Matching

Wave Propagation 1

- Note that the differential equations on the previous slide are coupled. We can therefore eliminate either V(z) or I(z).
- Doing so yields either of the differential equations:

$$\frac{d^2V(z)}{dz^2} - \gamma^2 V(z) = 0$$

$$\frac{d^2I(z)}{dz^2} - \gamma^2 I(z) = 0$$

where $\gamma = \alpha + j\beta = \sqrt{(R + j\varpi L)(G + j\varpi c)}$
is known as the propagation constant.

- α is the attenuation constant and represents the rate of decay of the wave amplitude with distance.
- β is the phase constant and represents the relative phase shift with respect to position along the line.

Wave Propagation 2

- The second order differential equations on the previous slide should be familiar as the wave equation, which has the well known solutions:

$$
\begin{aligned}
V(z) &= V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \\
I(z) &= I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}
\end{aligned}
$$

where $e^{-\gamma z}$ represents wave propagation in the +z direction and $e^{+\gamma z}$ represents wave propagation in the −z direction.

- Voltages and currents therefore propagate along the line as traveling waves.

Wave Propagation 3

- Given a transmission line with a propagating wave, we can define:
  - ✓ **The characteristic Impedance:**

$$Z_0 = \frac{R + j\omega L}{\gamma} = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \frac{V_0^+}{I_0^+} = \frac{-V_0^-}{I_0^-}$$

  - ✓ **The wavelength on the line:**

$$\lambda = \frac{2\pi}{\beta}$$

  - ✓ **The phase velocity**: The speed at which a constant phase point travels down the line, given by:

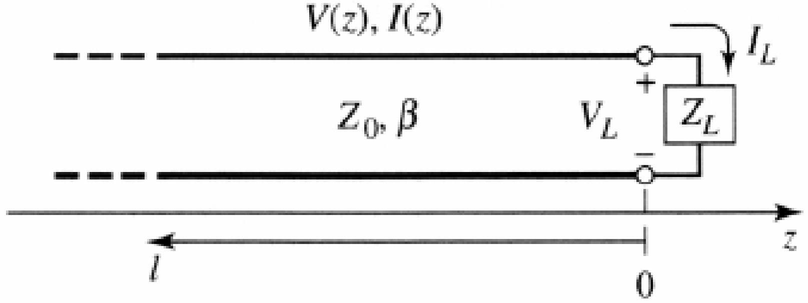
$$v_p = \frac{\omega}{\beta} = \lambda f$$

Lossless Lines

- The preceding general case can be simplified considerably for lossless lines, which can be used to approximate practical cases where the loss is often very small and can be neglected. In that case, **R=G=0**, and lossless line properties are:
  - ✓ **Propagation Constant:**
  $$\begin{aligned}\gamma &= j\omega\sqrt{LC}\\ \alpha &= 0\\ \beta &= \omega\sqrt{LC}\end{aligned}$$
  - ✓ **Characteristic Impedance:** $Z_0 = \sqrt{\frac{L}{C}}$
  - ✓ **Wavelength :** $\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{LC}}$
  - ✓ **Phase Velocity:** $v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$

Terminated Lines: General Case 1

A lossless transmission line terminated in a general load impedance Z_L experiences wave transmission and reflection.



The following properties are defined:

- **Reflection Coefficient**: the ratio of reflected to incident wave amplitude.

$$\Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

- **Return Loss**: the amount of incident power reflected from the load back towards the generator.

$$RL = -20 \log |\Gamma| \mathrm{dB}$$

Terminated Lines: General Case 2

- **Standing Wave Ratio**: incident and reflected waves on mismatched lines cause standing waves. The ratio of the maximum to minimum voltage amplitude is known as the standing wave ratio.

$$SWR = \frac{V_{max}}{V_{min}} = \frac{1+|\Gamma|}{1-|\Gamma|}$$

- **Input Impedance**: impedance looking into the line from the source varies with position. At a distance $l = -z$ from the load:

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l}$$

Terminated Lines: Short Circuit Case

If the line is short circuited, $Z_L = 0$ and the line has the following properties:

- Reflection coefficient, $\Gamma = -1$. The incident wave is completely reflected.
- Standing wave ratio is infinite.
- Voltage and current is given by:

$$V(Z) = -2jV_0^+ \sin \beta z$$
$$I(Z) = \frac{2V_0^+}{Z_0} \cos \beta z$$

- Input impedance:

$$Z_{in} = jZ_0 \tan \beta l$$

Terminated Lines: Open Circuit Case

If the line is open circuited, $Z_L = \infty$ and the line has the following properties:

- Reflection coefficient, $\Gamma = 1$. The incident wave is again completely reflected.
- Standing wave ratio is infinite.
- Voltage and current is given by:

$$V(Z) = -2V_0^+ \cos \beta z$$
$$I(Z) = \frac{2jV_0^+}{Z_0} \sin \beta z$$

- Input impedance:

$$Z_{in} = -jZ_0 \cot \beta l$$

Terminated Lines: Special Line Lengths

If a terminated transmission line has length, $l = \lambda/2$ or any multiple of $\lambda/2$:

- $Z_{in} = Z_L$
- The presence of the transmission line has no effect on input impedance.

If a terminated transmission line has length, $l = \lambda/4 + n\lambda/4$ for any $n \in \{1, 2, 3, ...\}$:

- $Z_{in} = \frac{Z_0^2}{Z_L}$
- The transmission line inversely transforms the load impedance.
- The line is referred to as a **quarter wave transformer**.
- It can be used for impedance matching.

RF and Microwave Systems

Transmission Lines

Smith Chart

Microwave Networks

Impedance Matching

Example: Transmission Line Calculations

A load impedance of $130 + j90\Omega$ terminates a 50Ω line that is 0.3λ long. Find:

1. Reflection coefficient at the load.

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(130 + j90) - 50}{(130 + j90) + 50} = 0.598\angle 21.8^\circ$$

2. Standing wave ratio.

$$SWR = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.598}{1 - 0.598} = 3.98$$

3. Return loss.

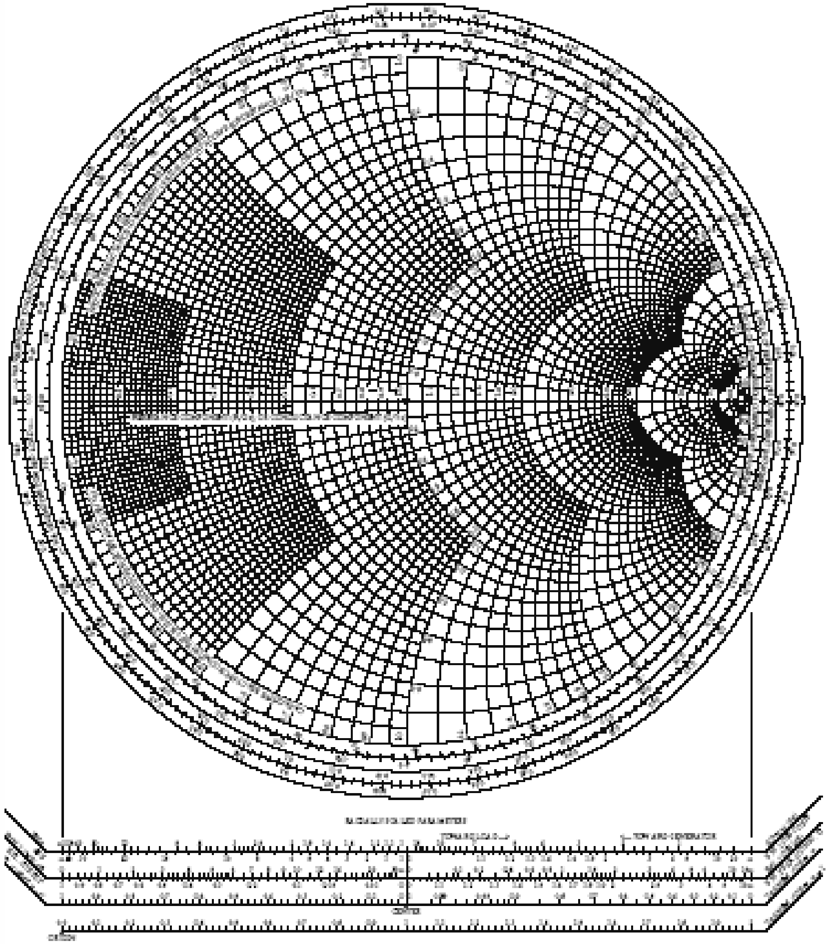
$$RL = -20 \log |\Gamma| = -20 \log(0.598) = 4.47\mathrm{dB}$$

4. Input impedance.

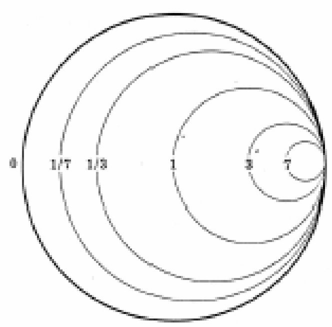
$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} = 12.75 + j5.8\Omega$$

The Smith Chart

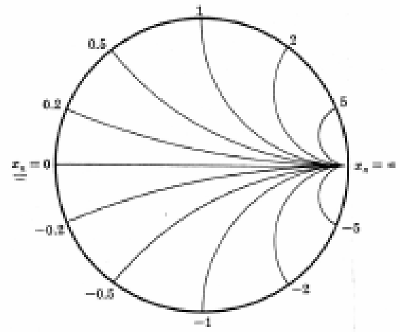
- The Smith chart is a graphical tool for solving transmission line problems.
- It represents a polar plot of voltage reflection coefficient.
- Magnitude (Γ) is measured as a radius from the centre of the chart.
- Angle $(-180 \leq \theta \leq 180)$ is measured from the right hand side diameter.
- Normalized impedances $(z = Z/Z_0)$ are represented on the chart.
- The Smith chart allows conversion between reflection coefficient, normalized impedance and admittance.



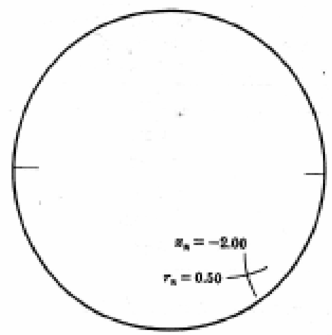
Reading the Smith Chart



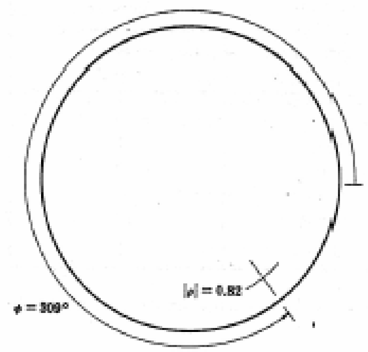
Constant resistance loci



Constant reactance loci



Normalized impedance



Reflection coefficient

Example: Using the Smith Chart with Impedance Values 1

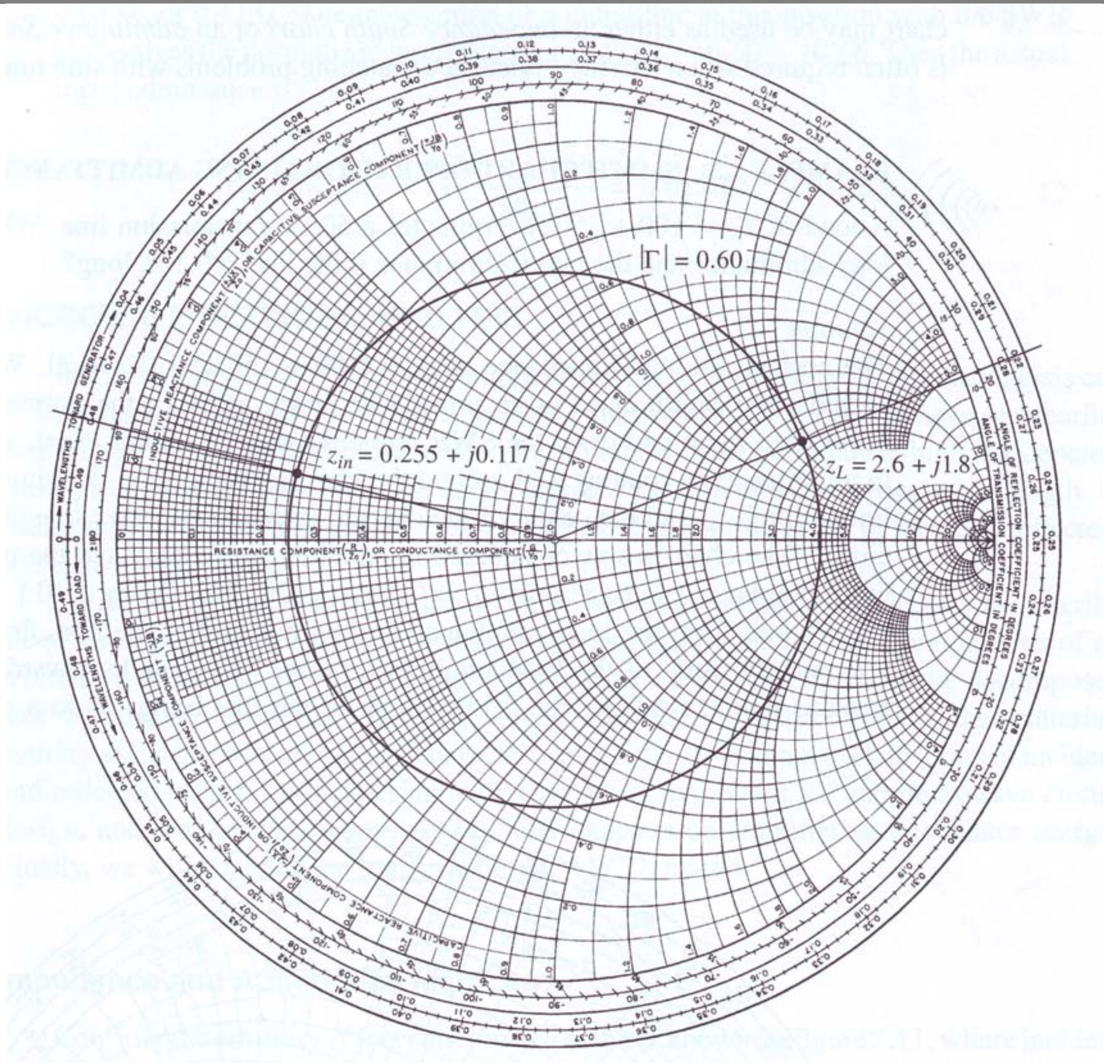
A load impedance of $130 + j90\Omega$ terminates a 50Ω line that is 0.3λ long. Find the reflection coefficient at the load, SWR, RL and Z_{in}

- Calculate and plot normalized impedance:

$$z_L = \frac{Z_L}{Z_0} = \frac{130 + j90}{50} = 2.6 + j1.9$$

  The radial distance from the centre of the chart to the point gives $|\Gamma| = 0.6$.
- The same radial measurement gives $SWR = 3.98$ and $RL = 4.4\text{dB}$ measured on the accompanying linear scale.
- The angle of the reflection coefficient can be read on the perimeter of the chart by drawing a radius passing through the impedance coordinate.
- Z_{in} is found by moving 0.3λ along the constant SWR circle. $Z_{in} = Z_0 z_{in} = 12.7 + j5.8\Omega$.

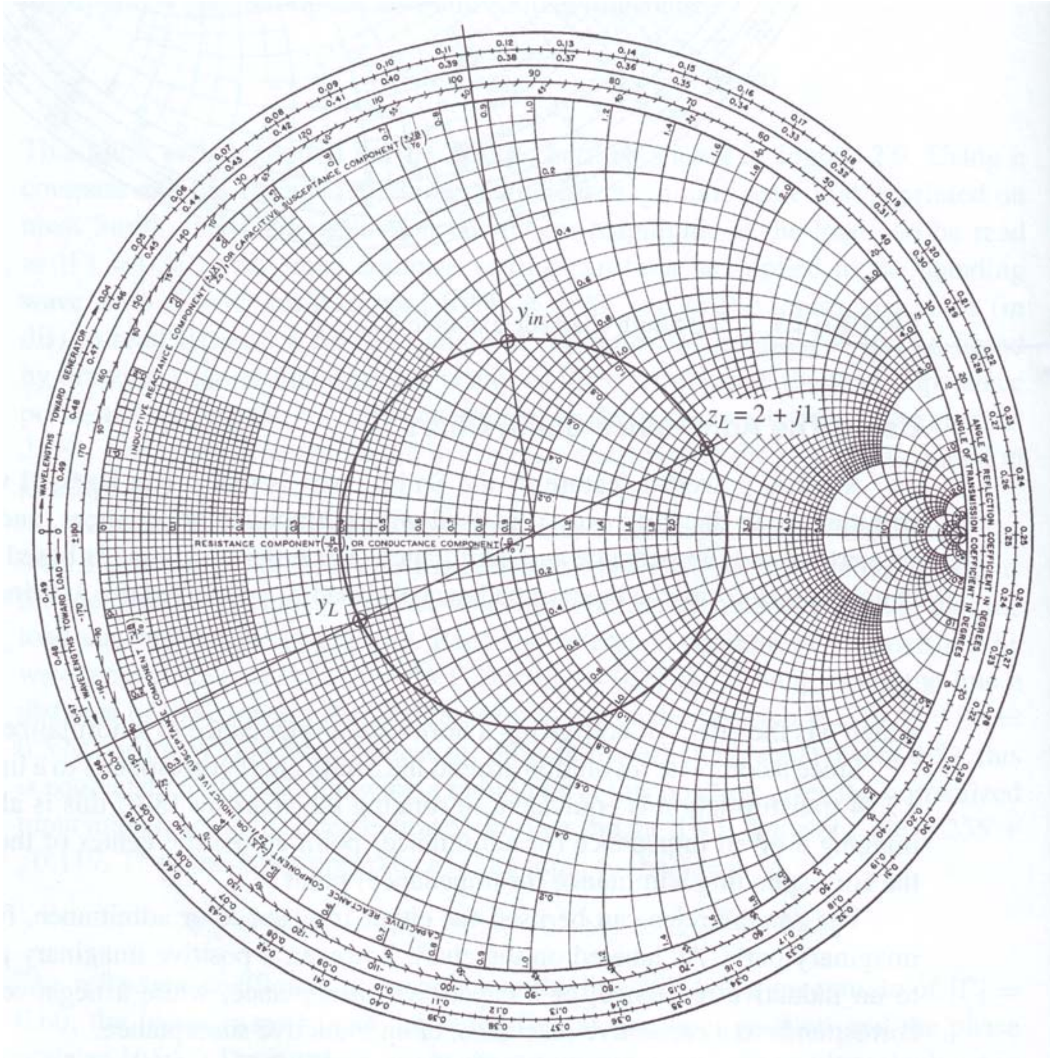
Example: Using the Smith Chart with Impedance Values 2



Using the Smith Chart with Admittance Values

- Any normalized impedance value can easily be converted to an admittance value using the Smith chart.
- Start by plotting the impedance coordinate.
- Draw the constant SWR circle passing through this point.
- Draw the diameter passing through the impedance coordinate.
- The normalized admittance can then be read as the point diametrically opposite to the normalized impedance.

Example: Using the Smith Chart with Admittance Values



Microwave Networks

Circuit analysis techniques, valid at low frequencies can be extended to RF and microwave circuits and networks. We will discuss:

- Impedance and Admittance matrices
- Scattering matrices
- Transmission (ABCD) matrices

Impedance and Admittance Matrix 1

The impedance (or admittance) matrix relates the total voltages and currents at the terminals of a N-port network.

- At each port, the total voltage and current consists of the sum of the incident and reflected waves.
- The $[Z]$ matrix can then be defined, relating the total voltages and currents at the ports as:

$$[V] = [Z][I]$$

- Element Z_{ij} of the impedance matrix is found by: (1) drive port j with current I_j and open circuit all other ports. (2) Measure the open circuit voltage at port i. (3) Z_{ij} is the transfer impedance between ports i and j:

$$Z_{ij} = \left.\frac{V_i}{I_j}\right|_{I_k = 0 \text{for} k \neq j}$$

- The admittance matrix $[Y]$ can similarly be defined by: $[I] = [Y][V]$.

Impedance and Admittance Matrix 2

- The matrix elements are complex in general.
- The impedance or admittance matrix is symmetric if the network is reciprocal. This means that the network contains no nonreciprocal media such as ferrites, active devices or plasmas.
- It can be shown that a lossless network dissipates no real power and that all Z_{ij} and Y_{ij} elements are purely imaginary.

Scattering Matrix 1

- At high frequencies, total voltages and currents can be difficult to measure.
- The scattering matrix is another way to characterize a microwave network.
- It relates incident(V^+) and reflected (V^-) voltage waves at the terminals of a N-port network:

$$[V^-] = [S][V^+]$$

- Element S_{ij} of the scattering matrix is found by: (1) Drive port j with an incident wave V_j^+ (2) Measure the reflected wave amplitude V_i^- at port i, while (3) All other ports are terminated in matched loads to prevent reflections from those ports.

$$S_{ij} = \left. \frac{V_i^-}{V_j^+} \right|_{V_k^+ = 0 \mathrm{for} k \neq j}$$

Scattering Matrix 2

- S_{ii} represents the reflection coefficient looking into port i.
- We can convert beween the scattering matrix and the impedance matrix as follows:

$$[S] = ([Z] + [U])^{-1}([Z] - [U])$$

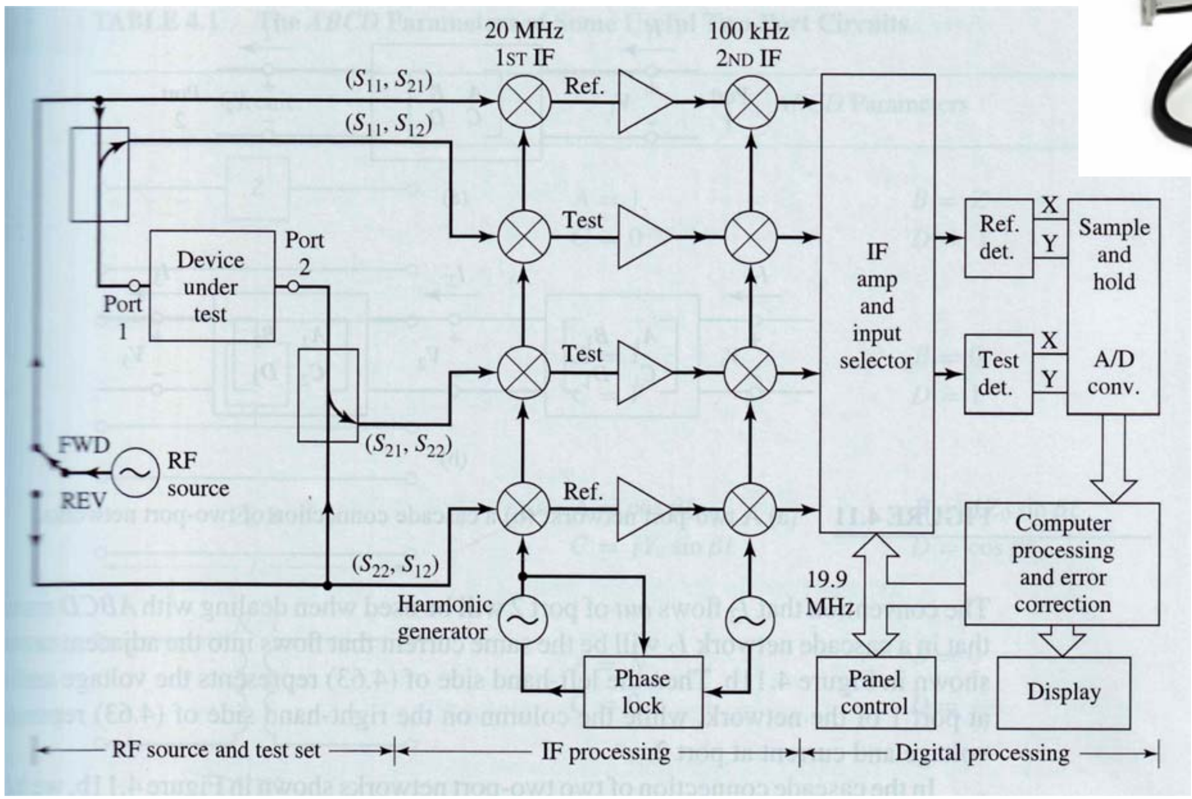
$$[Z] = ([U] - [S])^{-1}([U] + [S])$$

where $[U]$ is the identity matrix.

- $[S]$ is symmetric for reciprocal networks and unitary for lossless networks.
- Scattering matrix elements are measured using a network analyser.

Measurement of S parameters

- **Network Analyzer**
  - ✓ 2 port Network (S_{11},S_{12},S_{21},S_{22})
  - ✓ Measurements require a calibration process



Transmission (ABCD) Matrix

- 2 port networks are common in microwave circuits.
- The 2x2 transmission or *ABCD* matrix relates total voltages and currents by:

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

- The *ABCD* matrix of a cascade connection is the product of the two *ABCD* matrices.
- One can convert between impedance and transmission matrices using:

$$A = \frac{Z_{11}}{Z_{21}} \quad B = \frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{21}}$$

$$C = \frac{1}{Z_{21}} \quad D = \frac{Z_{22}}{Z_{21}}$$

- Reciprocal networks have $AD - BC = 1$.

Impedance Matching

Impedance matching is the process on inserting a network between the generator and the load, so that the input impedance matches the characteristic impedance. Reflections on the line are then minimized and power transfer maximized. We will discuss:

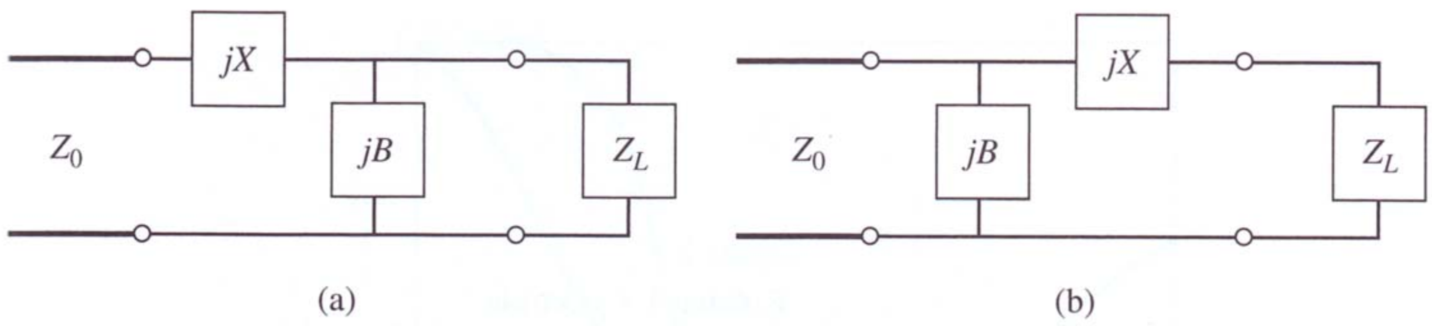
- Quarter wave transformer
- L-section matching
- Single stub tuning

Quarter Wave Transformer

- Insert a $\lambda/4$ length of line of impedance $Z_1 = \sqrt{Z_0 Z_L}$ between the line and the load.
- The input impedance will then match the line characteristic impedance.
- Disadvantage: Only works well for relatively narrowband matching.

L-Section

- The matching network uses 2 reactive circuit components in a series and shunt connection with the load to provide matching.
- If z_L is inside $1 + jx$ circle, network (a) is used. If z_L is outside $1 + jx$ circle, network (b) is used.
- Matching problem can be solved using the Smith chart.



Single Stub Tuning

- Uses a single open circuited or short circuited line length connected in either series or paralell at a distance d from the load.
- Tuning requires one to determin the distance from the load d and shunt susceptance (or series reactance) provided by the stub to obtain a match.

Shunt Stub

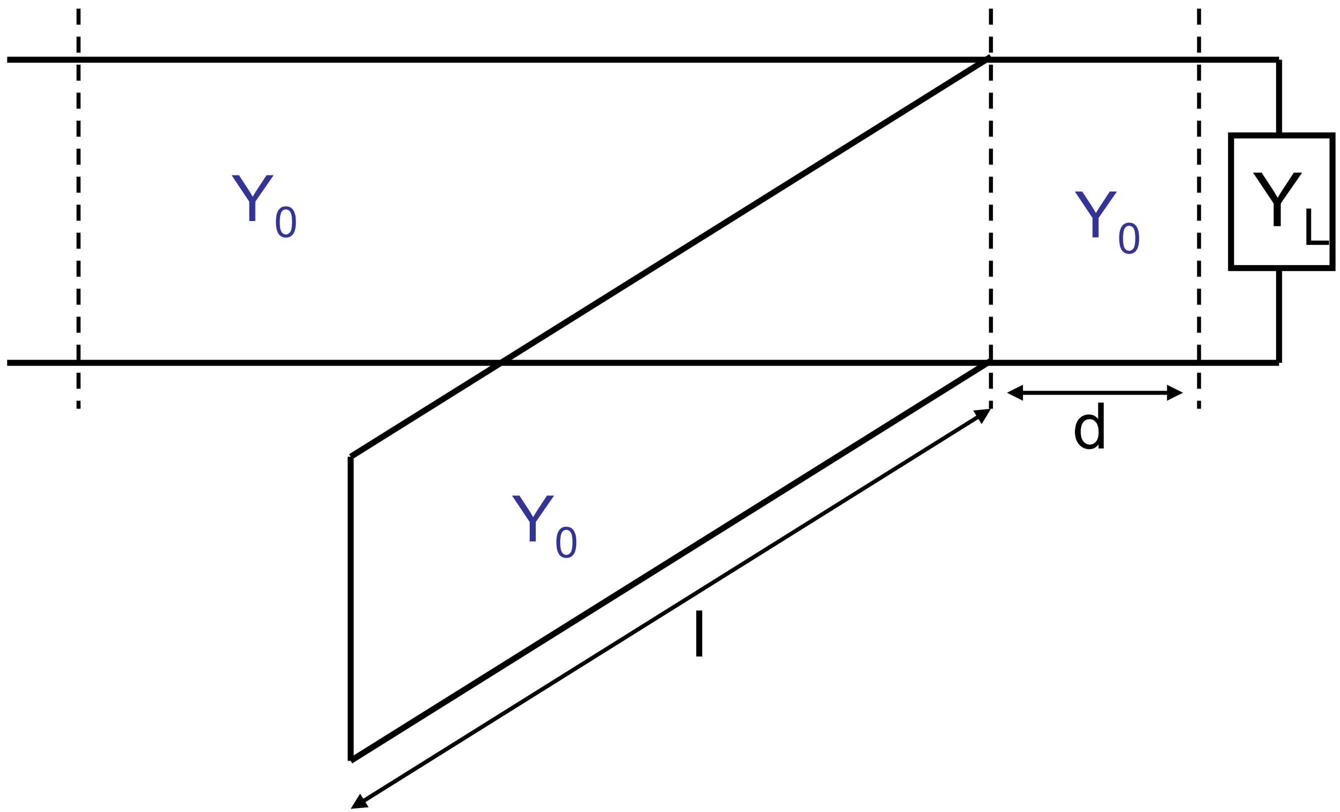
- Choose d such that admittance Y looking into the line is $Y_0 + jB$ where $Y_0 = 1/Z_0$.
- The stub susceptance is then chosen to be $-jB$ resulting in a match.

Series Stub

- Choose d such that impedance Z looking into the line is $Z_0 + jX$.
- The stub reactance is then chosen to be $-jX$ resulting in a match.

Single Stub Tuning

- **Shunt stub (Y = 1/Z)**

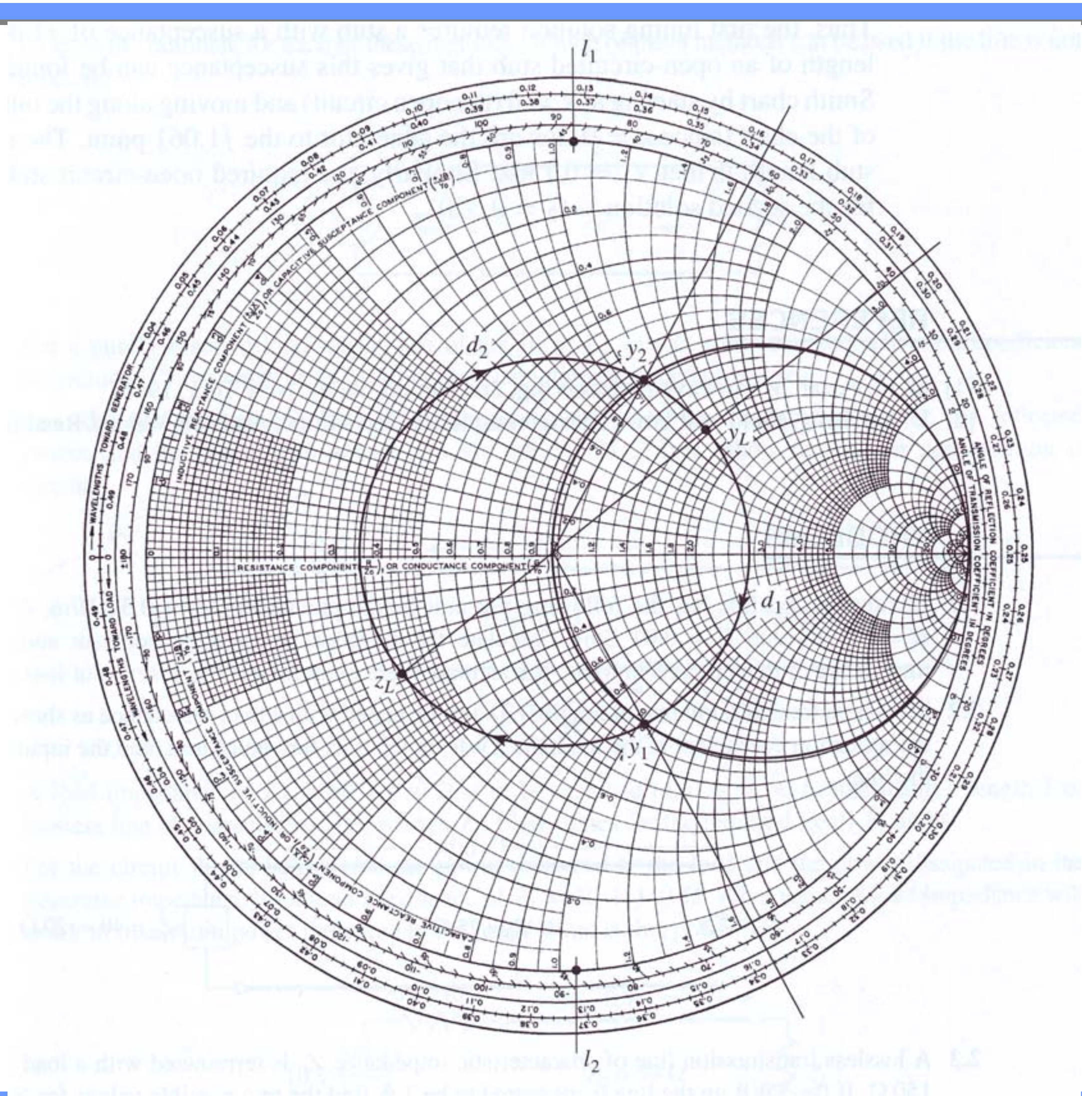


Example: Single Stub Shunt Tuning 1

A 50Ω line has a load of $Z_L = 20 - j15\Omega$. Design a single stub shunt network to match the line.

- Plot normalized load impedance $z_L = 0.4 - j0.3$ on the Smith chart.
- Find the corresponding normalized load admittance by drawing the SWR circle and reading the point diametrically opposite to z_L. $y_L = 1.6 + j1.2$.
- We want to find d such that normalized admittance looking into the line is equal to 1, so we find the intersection if the SWR circle with the $1 + jb$ circle and read the wavelengths towards generator scale on the periphery of the chart to determine d.
- The 2 intersections give $d_1 = 0.14\lambda$ and $d_2 = 0.468\lambda$.
- At these intersection points we have $y_1 = 1 - j1.061$ and $y_2 = 1 + j1.061$, which gives 2 valid solutions.
- The length of the stub for the first solution can be read off the Smith chart by starting at $y = 0$ and moving towards the generator to the $j1.061$ point. We find $l_1 = 0.13\lambda$.

Example: Single Stub Shunt Tuning 2



Drill problems

Answer the following questions, found at the end of Chapter 2 of "Microwave and RF Design of Wireless Systems" by Pozar.

1. Problem 2.4
2. Problem 2.5
3. Problem 2.15
4. Problem 2.24
5. Problem 2.25